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M460 ***Essay***

Damage Evolution

Introduction:

Damage evolution looks at the opening and closing of micro defects in a material that impact the integrity of the material. Damage evolution can describe the material properties of material undergoing ductal damage, fatigue damage and creep damage. When looking at a stress-strain curve, damage evolution describes a material's downward trend after reaching its ultimate tensile strength. While other failures look at defects in materials, damage evolution looks at how defects appear in a defect-free material and how the ultimate result is a material's failure.

Current approach:

Damage evolution can be broken up into three primary steps initiation, growth, and coalescence. During the initiation stage, microscopic voids or discontinuities appear in the material distributed throughout the material based on stress concentrations. During the growth stage, these discontinuities increase in size. And finally, in the coalescence phase, discontinuities combine to become a macroscopic feature that leads to the failure of the material. The image below shows this damage evolution process occurring in a tensile test.

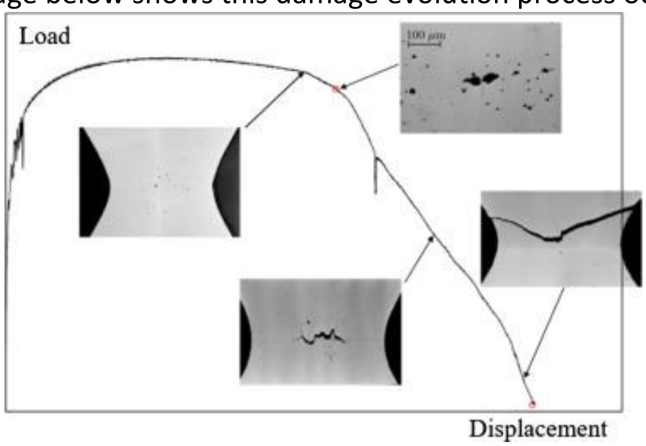

Figure 1: Damage evolution in a tensile test [1]

To help understand this model, we can take some values from a standard tensile strength curve. ε_0^{pl} is when material starts accumulating damage at the σ_{y0}, which is the material's ultimate strength. After that point, the stress decreases due to the damage. σ_R is the rupture stress of the material, which is the stress at which fracture occurs, at which the stress goes to zero.

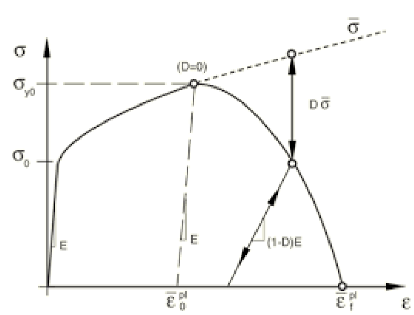

Figure 2: Stress-Strain Curve of Abaqus damage model [2]

From this graph, we can see that rupture occurs when $D=D_c$ and that can be calculated using the formula below. Once the damage has reached this point, it will result in the part failing.

$$D_c = 1 - \frac{\sigma_R}{\sigma_u}$$

To track how much damage in a material sample the Young's modulus can be measured at several points through a tensile test. From the young's modulus the damage can be calculated using the following formula.

$$D = 1 - \frac{\tilde{E}}{E}.$$

Damage does not occur at all points, only when the stored plastic energy exceeds the plastic energy at which damage occurs. ω_S is the total plastic energy stored in a material. ω_D is the minimum stored plastic energy for damage to occur. There are 2 different methods used to integrate the plastic energy shown in the plot below, but it does not matter which you use as long as you use the same one to calculate ω_S and ω_D.

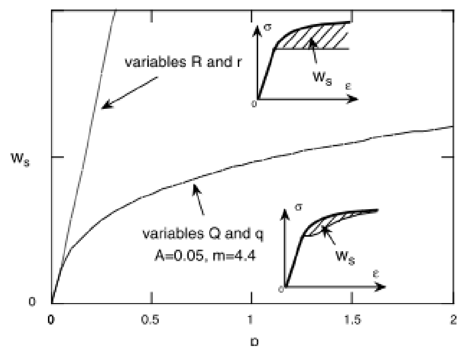

Figure 3: Plastic strain energy vs plastic strain [3]

Both are functions of total plastic strain, which means that from ω_D, you can calculate an equivalent strain p_D at which damage would occur. For monotonic tests, this is simply ε_{pD}. Still, for cyclical loading, it is ε_{pD} multiplied by a correction factor that is a function of the fatigue limit of the material, the ultimate tensile stress and the max and min stress.

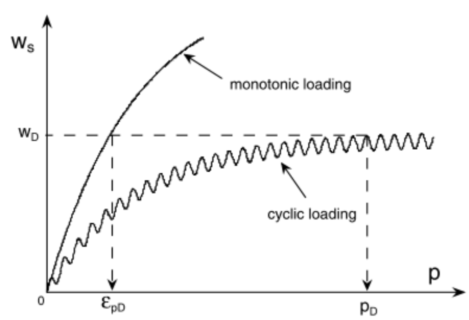

Figure 4: Plot of plastic energy vs plastic strain [3]

$$p_D = \epsilon_{pD} \text{ for monotonic loading,}$$

$$p_D = \epsilon_{pD} \left(\frac{\sigma_u - \sigma_f^\infty}{\frac{\sigma_{eq\,max} + \sigma_{eq\,min}}{2} - \sigma_f^\infty} \right)^m \text{ for cyclic loading}$$

From this we understand when damage will occur, but we still have not calculated the rate at which it will occur. F_D is the dissipative damage potential function which is the energy stored in a material that can result in damage. Y is the energy density release rate, density times the derivative of Gibbs free energy by damage.

$$\boxed{Y = \rho\frac{\partial \psi^\star}{\partial D} = \frac{\tilde{\sigma}_{\mathrm{eq}}^2 R_\nu}{2E},}$$

$\dot{\lambda}$ is the plastic multiplier, a material property describing the rate at which plastic strain occurs. This all cumulates in the damage rate for a material, able to be calculated using the formula below.

$$\dot{D} = \dot{\lambda}\frac{\partial F_{\mathrm{D}}}{\partial Y} \qquad \text{if} \quad p > p_{\mathrm{D}} \quad \text{or} \quad \max w_{\mathrm{s}} > w_{\mathrm{D}}$$

Since F_D is a function of Y, we can experimentally determine a formula for F_D that fits data with the form below.

$$F_{\mathrm{D}} = \frac{S}{(s+1)(1-D)}\left(\frac{Y}{S}\right)^{s+1}$$

When substituted into the damage evolution equation, we get the following formula.

$$\dot{D} = \left(\frac{Y}{S}\right)^s \dot{p}$$

$\dot{p}$ is the plastic strain rate of the material that can be calculated from Ductile, creep or fatigue plastic strain rates. Y can be estimated using the formula below which gives us a formula for damage that can be fit

$$\begin{cases} Y = \dfrac{\tilde{\sigma}_{\mathrm{eq}}^2 R_\nu}{2E} \\ R_\nu = \frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\dfrac{\sigma_{\mathrm{H}}}{\sigma_{\mathrm{eq}}}\right)^2 \end{cases}$$

Examples:

Plasticity

Plasticity is the easiest to model because $\dot{p}$ is equal to the plastic strain of the material which is equal to the strain rate of the test due to the fast testing window of a tensile test. [3]

$$\dot{D} = \left(\frac{Y}{S}\right)^s \dot{p} = \left(\frac{\sigma_{\mathrm{u}}^2}{2ES}\right)^s |\dot{\epsilon}_{\mathrm{p}}| \qquad \text{if} \quad \epsilon_{\mathrm{p}} > \epsilon_{\mathrm{pD}}$$

The plot below shows a tensile test where data for creating this model can be gathered. As necking is a completely separate phenomenon to damage, the effect of strain can be removed through correction factors. [3]

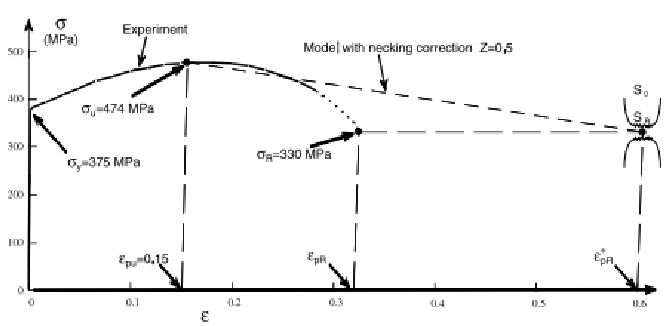

Figure 5: Stress-strain curve with damage model [3]

Creep

We can apply this model to fatigue using the same one we used for plastic deformation. From the above test, we have calculated the model parameters above, and the only thing we need to calculate is the plastic strain rate due to creep.

$$\dot{D} = \left(\frac{\sigma^2}{2ES(1-D)^2}\right)^s |\dot{\epsilon}_p| \qquad \text{if} \quad p > \epsilon_{pD} \quad \dot{\boldsymbol{\epsilon}}^p = \frac{3}{2}\frac{\boldsymbol{\sigma}^D}{\sigma_{eq}}\dot{p},$$

The plot below shows that this model is applied to an Abaqus model to simulate the damage at different radiuses on a sphere compared to an analytical.

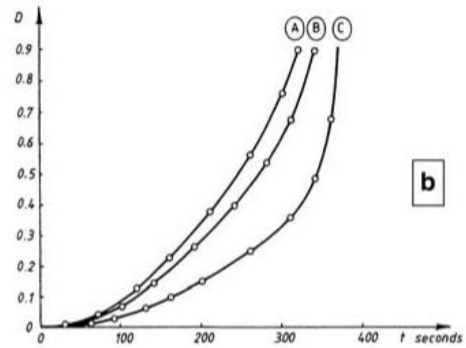

Figure 6: Damage during a pure creep test [3]

Conclusions:

Damage evolution is a method of tracking material damage that can account for various loading conditions and causes. Damage results in a change in the material properties primarily through decreasing Young's modulus of the material. Damage evolution has three significant stages initiation, growth, and coalescence, where the rate of formation changes. Once damage reaches a critical value it results in the complete failure of the material through rupture. Any plastic deformation causes damage through creep, fatigue, or plastic deformation. The model discussed in this essay shows how to track this damage rate through different loading conditions and the damage of a material can be determined through integration.

References

[1] H. Li and M. Fu, "Chapter 3 - Damage Evolution and Ductile Fracture," in *Deformation-Based Processing of Materials*, H. Li and M. Fu, Eds., Elsevier, 2019, pp. 85–136. doi: 10.1016/B978-0-12-814381-0.00003-0.

[2] "ABAQUS Analysis User's Manual (v6.6)." https://classes.engineering.wustl.edu/2009/spring/mase5513/abaqus/docs/v6.6/books/usb/default.htm?startat=pt05ch18s02abm21.html (accessed Apr. 12, 2023).

[3] *Engineering Damage Mechanics*. Berlin/Heidelberg: Springer-Verlag, 2005. doi: 10.1007/b138882.